

Causality Constraint on Einstein-Weyl Gravity

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ABSTRACT: We explore, in the context of AdS/CFT correspondence, the causality constraints on the Noncritical Einstein-Weyl (NEW) gravity model in five dimensions. The scalar and shear channels are considered as small metric perturbations around an AdS black brane background. Our results show that causality analysis on the propagation of these two channels imposes a new bound on the coupling of the Weyl-squared terms in the NEW gravity. This new bound imposes more stringent restrictions than those of the tachyon-free condition, improving predictive power of the theory.

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1 Introduction

Recently much attention has been paid to topologically massive gravity[1] in three dimensions. The model contains a massive spin-2 mode in addition to the usual massless graviton. The situations are changed after a special Chern-Simons coupling has been chosen. The extra massive spin-2 mode becomes massless, and the model reduces to the standard Einstein gravity after truncating out the logarithmic fall-off modes by imposing an appropriate boundary condition [2]. Therefore, the model is a toy model for a quantum theory of gravity due to its possibly well-controlled UV behaviour.

A recent generalization of this “critical” theory to four [3] and higher dimensions [4] results in a model known as “critical gravity”. This is a theory of gravity which includes Einstein-Hilbert term, cosmological constant and Weyl-squared terms in its action. In this model a special value (critical point) of the coupling of the Weyl-squared terms is chosen so that the massive spin-2 mode becomes a logarithmic mode. The new model presents many remarkable properties: (i) The model is perturbatively renormalizable since the action includes curvature-squared corrections as studied in detail in [5]; (ii) Due to the special coupling of Weyl-squared terms, the model contains a massless graviton and a logarithmic mode, which distinguishes it from most of models with higher derivative terms;

(iii) The massless graviton has zero on-shell energy, making the theory trivial. More recent discussion on this model can be found in a lot of literature [6] \sim [27].

Recent progress[17, 28] shows that by abounding the severe constraint imposed on the value of the coupling, one can generalize critical gravity to the so-called Noncritical Einstein-Weyl (NEW) gravity. This is a wider class of Weyl-squared extensions to Einstein gravity with cosmological constant. The value of the Weyl-squared coupling in this model is no longer fixed at the critical point, instead it relaxes to a certain range with which the massive mode in the AdS background is still tachyon-free. While this mode is still ghostlike, it can be truncated by imposing appropriate boundary conditions since this massive mode falls off more slowly than the massless graviton. Then only the massless graviton survives in this theory and this mode, different from critical gravity, has the positive excitation energy.

On the other hand a recent result obtained from AdS/CFT correspondence [29] implies that inclusion of higher-order curvature terms in the Einstein-Hilbert action may possibly introduce the superluminal propagation of boundary perturbations [30] \sim [35]. In order to preserve causality at the boundary, constraints have to be imposed on the couplings of the higher-order terms. It was first realized by Horman and Maldacena in [36] and then by Buchel and Myers in [34] that the causality constraints are closely related to the constraints¹ imposed on the ratio of the central charges, a/c , in the dual CFT.

Inspired by the recent observation in [37], AdS/CFT correspondence of NEW gravity was discussed in [38]. In this paper we would like to consider the causality constraints, in the context of the AdS/CFT correspondence, on the couplings of the Weyl-squared terms. Our calculation are performed in the gravity side, by studying their metric fluctuations around an AdS black brane background. Our results show that to preserve the causality at the boundary, new bounds on the Weyl-squared couplings are introduced.

The rest of the paper is organized as follows. In the next section we give a brief review on the Einstein-Weyl gravity, and the bounds on the Weyl-squared coupling are given by requiring that the massive mode is tachyon-free. In section three we find an approximate black brane solution to the NEW gravity model. In section four we consider a small metric fluctuations around the black brane, and a set of linearized wave equations are obtained for tensor-type and vector-type perturbations. In section five causality constraints on the couplings are discussed for both the scalar channel and shear channel. Conclusions and discussions are presented in the last section.

2 Einstein-Weyl Gravity: A Brief Review

In this section we would like to give a brief review on Einstein-Weyl conformal gravity. The starting point is a gravitational action containing higher curvature correction terms. We generally pay much attention to the quadratic corrections only. The most general quadratic

¹This constraints come from the requirement that the energy measured in calorimeters of a collider physics experiment should be positive[36].

gravity model with a cosmological constant is given by

$$I = \int d^D x \sqrt{-g} \left[\frac{1}{\kappa} (R - 2\Lambda) + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} + \gamma \mathcal{L}_{GB} \right], \quad (2.1)$$

where $\kappa = 16\pi G$ and \mathcal{L}_{GB} is the Gauss-Bonnet term

$$\mathcal{L}_{GB} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2.$$

This term can be generated from the low energy effective theory of heterotic and bosonic string theory, and it is a topological invariant in four dimensions.

It is straightforward to show that the equations of motion that follow from the action (2.1) are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu}^{\text{eff}}, \quad (2.2)$$

where

$$\begin{aligned} T_{\mu\nu}^{\text{eff}} = & \frac{\alpha}{2} (g_{\mu\nu}R^2 - 4RR_{\mu\nu} + 4\nabla_\nu \nabla_\mu R - 4g_{\mu\nu}\square R) + \\ & + \frac{\beta}{2} (g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} + 4\nabla_\alpha \nabla_{(\nu} R_{\mu)}^\alpha - 2\square R_{\mu\nu} - g_{\mu\nu}\square R - 4R_\mu^\alpha R_{\alpha\nu}) + \\ & + \frac{\gamma}{2} (g_{\mu\nu}\mathcal{L}_{GB} - 4R_{\mu\alpha\beta\gamma}R_\nu^{\alpha\beta\gamma} - 4RR_{\mu\nu} + 8R_\mu^\alpha R_{\alpha\nu} + 8R^{\alpha\beta}R_{\mu\alpha\nu\beta}), \end{aligned} \quad (2.3)$$

with ∇ the covariant derivative and $\square \equiv \nabla^2$ the d'Alembert operator. For Einstein-Weyl gravity, we have

$$\alpha = -\frac{D(D-3)\gamma}{(D-1)(D-2)}, \quad \beta = \frac{4(D-3)\gamma}{D-2},$$

and the action reduces to

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{\kappa} (R - 2\Lambda) + \gamma C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right], \quad (2.4)$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl conformal tensor and satisfies

$$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - \frac{4}{D-2} R_{\mu\nu} R^{\mu\nu} + \frac{2}{(D-1)(D-2)} R^2. \quad (2.5)$$

The equation of motion following this action then becomes

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu}^{\text{Weyl}}, \quad (2.6)$$

where

$$\begin{aligned} T_{\mu\nu}^{\text{Weyl}} = & \gamma \left\{ -\frac{1}{(D-1)(D-2)} R(4R_{\mu\nu} - g_{\mu\nu}R) - \frac{2}{D-2} [g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} - 2(D-2)R_\mu^\rho R_{\rho\nu}] \right. \\ & + \frac{2(D-3)}{(D-1)(D-2)} [g_{\mu\nu}\square R + (D-2)\nabla_\mu \nabla_\nu R] - \frac{4(D-3)}{D-2} \square R_{\mu\nu} \\ & \left. + \frac{1}{2}g_{\mu\nu}R_{\rho\sigma\lambda\kappa}R^{\rho\sigma\lambda\kappa} - \frac{4(D-4)}{D-2} R^{\rho\sigma} R_{\mu\rho\nu\sigma} - 2R_{\mu\rho\sigma\gamma}R_\nu^{\rho\sigma\gamma} \right\}. \end{aligned} \quad (2.7)$$

One of the biggest issues with Einstein-Weyl gravity theory, just like most of the higher derivative theories, is the presence of ghosts, which point to instabilities of the quantum version of the theory. To see this clearly, we note that the Einstein-Weyl theory admits AdS spacetime as the vacuum solution $\bar{g}_{\mu\nu}$ of the field equations, and we have

$$\bar{R} = \frac{2D}{D-2}\Lambda, \quad \bar{R}_{\mu\nu} = \frac{2\Lambda}{D-2}\bar{g}_{\mu\nu}, \quad \bar{R}_{\mu\nu\rho\sigma} = \frac{2\Lambda}{(D-1)(D-2)}(\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}). \quad (2.8)$$

By considering the metric fluctuations around this AdS vacuum $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, we obtained linearized equation of motion

$$\left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)} - M^2\right) \left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)}\right) h_{\mu\nu} = 0, \quad (2.9)$$

where $\bar{\square} = \bar{\nabla}^2$ with $\bar{\nabla}$ the covariant derivative with respect to $\bar{g}_{\mu\nu}$ and M^2 in (2.9) is defined as

$$M^2 \equiv -\frac{D-2}{4(D-3)} \left(\frac{1}{\kappa\gamma} - \frac{8(D-3)\Lambda}{(D-1)(D-2)} \right) = m^2 - \frac{D-2}{L^2}, \quad (2.10)$$

where L is the radius of the AdS space and

$$m^2 \equiv -\frac{D-2}{4(D-3)\kappa\gamma}.$$

Note that in deriving (2.9) the transverse and traceless gauge, which is consistent with the equation of motion, has been used

$$\bar{\nabla}^\mu h_{\mu\nu} = 0, \quad h = 0. \quad (2.11)$$

As a consequence, the fourth order differential equation (2.9) of the Einstein-Weyl conformal gravity consists of a massless mode $h_{\mu\nu}^{(m)}$ and a massive mode $h_{\mu\nu}^{(M)}$ which satisfy, respectively, the wave equations

$$\left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)}\right) h_{\mu\nu}^{(m)} = 0, \quad \left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)} - M^2\right) h_{\mu\nu}^{(M)} = 0. \quad (2.12)$$

In a special case where $M^2 = 0$, i.e.

$$m^2 = \frac{D-2}{L^2} \quad \text{or} \quad \gamma = -\frac{L^2}{4(D-3)\kappa}, \quad (2.13)$$

the two linearized equations of motion degenerate. This case is generally called critical gravity[3, 4]. The second mode solution in this case is no longer massive, instead, it becomes a logarithmic mode in the sense that this mode $h_{\mu\nu}^{(L)}$ satisfies a quadratic differential equation,

$$\left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)}\right)^2 h_{\mu\nu}^{(L)} = 0. \quad (2.14)$$

It is generally believed that the logarithmic mode can be truncated by imposing a suitable boundary condition at the conformal boundary due to the fact that the logarithmic mode falls off more slowly than massless mode as it approaches the boundary. Recent progress[3,

[27] shows, however, that conformal gravity at the critical point seems to be trivial since the massless mode has zero energy of excitations. As a consequence, particular attention has been paid to the case with $-\frac{(D-3)^2}{4L^2} \leq m^2 < \frac{D-2}{L^2}$ where the theory is still tachyon-free since it is above the Breitenlohner-Freedman bound[39]. In this case, we notice that the massive mode falls off less rapidly at the boundary than massless mode since $M^2 < 0$. Therefore, one can truncate the massive mode by imposing suitable boundary condition as before and the massless mode with non-zero excitation energy survives only. In what follows, we mainly focus on the conformal gravity with this coupling, i.e.,

$$-\frac{L^2}{4(D-3)\kappa} < \gamma \leq \frac{(D-2)L^2}{(D-3)^3\kappa}. \quad (2.15)$$

This is often referred to as the “tachyon-free” condition.

3 AdS Black Brane in Conformal Gravity

In this section we would like to find an AdS black brane solution to the Einstein-Weyl conformal gravity. Owing to the fact that the field equations of conformal gravity are a set of fourth order differential equations, it is generally quite hard to obtain their exact solutions. There are some exact solutions of pure conformal gravity (i.e., the limited case of Einstein-Weyl gravity when γ goes to infinity) in four dimensions in the literatures, say [40]. However, it is very difficult to obtain an exact solution to Einstein-Weyl gravity with the presence of Ricci scalar and cosmological constant in any dimension. For simplicity, we find our AdS black brane solution approximately.

At the beginning, we note that for the following Einstein-Hilbert action with negative cosmological constant

$$I = \frac{1}{\kappa} \int d^D x \sqrt{-g} (R - 2\Lambda), \quad (3.1)$$

there is an AdS black brane solution in Poincaré coordinates

$$ds^2 = \frac{1}{z^2} \left(\frac{-f(z)dt^2 + d\vec{x}^2}{L^2} + \frac{L^2 dz^2}{f(z)} \right), \quad (3.2)$$

where

$$f(z) = 1 - \left(\frac{z}{z_0} \right)^{D-1}, \quad (3.3)$$

with z_0 the horizon of the black brane.

The corrected black brane metric of the Einstein-Weyl gravity can be obtained by noting that all terms in $T_{\mu\nu}^{Weyl}$ are higher order corrections as shown in (2.7). Hence one can calculate $T_{\mu\nu}^{Weyl}$ by substituting the unperturbed metric (3.2). A perturbatively approximate solution of the field equation (2.6) calculated in this way is given by (for details, see Appendix B in [30])

$$ds^2 = \frac{1}{z^2} \left(\frac{-N^2 f(z)dt^2 + d\vec{x}^2}{L^2} + \frac{L^2 dz^2}{f(z)} \right) \quad (3.4)$$

where N^2 is a constant whose value will be determined later and

$$f(z) = 1 - \left(\frac{z}{z_0}\right)^{D-1} + \lambda \left(\frac{z}{z_0}\right)^{2(D-1)} \quad (3.5)$$

with λ a constant

$$\lambda = \frac{(D-3)(D-4)}{L^2} \kappa \gamma.$$

If we change the variable z to u as

$$u = \left(\frac{z}{z_0}\right)^{\frac{D-1}{2}},$$

we obtain the following metric

$$ds^2 = \frac{-N^2 f(u) dt^2 + d\vec{x}^2}{L^2 z_0^2 u^{\frac{4}{D-1}}} + \frac{4L^2}{(D-1)^2} \frac{du^2}{u^2 f(u)}, \quad (3.6)$$

where $f(u)$ now becomes

$$f(u) = 1 - u^2 + \lambda u^4. \quad (3.7)$$

In these coordinates, $u = 0$ is the boundary of the AdS space, and the horizon of the black brane is at

$$u_H \simeq 1 + \frac{\lambda}{2} \quad (3.8)$$

Taking the limit $\lambda \rightarrow 0$, the solution corresponds to Schwarzschild-AdS spacetime. The causality analysis in this background has been well investigated in [31].

To determine the value of the constant N^2 in the metric (3.6), we notice that the space-time geometry of the background at the boundary would reduce to flat Minkowski metric conformally, i.e. $ds^2 \propto -c^2 dt^2 + d\vec{x}^2$. On the boundary $r \rightarrow \infty$, we have

$$N^2 f(u \rightarrow 0) \rightarrow 1,$$

so that N^2 is found to be

$$N^2 = 1, \quad (3.9)$$

by specifying the boundary speed of light to be unity $c = 1$. Therefore, the metric of the black brane in (3.6) is now of the form

$$ds^2 = \frac{-f(u) dt^2 + d\vec{x}^2}{L^2 z_0^2 u^{\frac{4}{D-1}}} + \frac{4L^2}{(D-1)^2} \frac{du^2}{u^2 f(u)}. \quad (3.10)$$

This is the AdS black brane solution which will be mainly considered in the following.

4 Linearized Wave Equation in the Conformal AdS Black Brane Background

To see the propagation of the transverse graviton, in this section we shall consider small metric fluctuations $h_{\mu\nu}$ around the AdS black brane background (3.10)

$$g_{\mu\nu} \equiv g_{\mu\nu}^{(0)} + h_{\mu\nu}, \quad (4.1)$$

where the background metric $g_{\mu\nu}^{(0)}$ is given in (3.10). To the first order of the metric perturbation, one can define an inverse metric as

$$g^{\mu\nu} = g^{(0)\mu\nu} - h^{\mu\nu} + \mathcal{O}(h^2), \quad (4.2)$$

and the indices are raised and lowered by using these background metric $g_{\mu\nu}^{(0)}$ and $g^{(0)\mu\nu}$. We also denote a trace part of the metric and a field strength for the perturbative parts as $h \equiv g^{(0)\mu\nu} h_{\mu\nu}$.

Now we would like to think of a linearized theory of the symmetric tensor field $h_{\mu\nu}$ propagating in the conformal AdS black brane background. To the first order of $h_{\mu\nu}$, the Einstein equation (2.6) can be written as

$$R_{\mu\nu}^{(1)} - \frac{1}{2}g_{\mu\nu}^{(0)}R^{(1)} - \frac{1}{2}h_{\mu\nu}R^{(0)} + h_{\mu\nu}\Lambda = \kappa T_{\mu\nu}^{(1)} \quad (4.3)$$

In the expression above, the scalar curvature $R^{(0)}$ is constructed by using the background metric $g_{\mu\nu}^{(0)}$ and the following tensors are newly defined:

$$R_{\mu\nu}^{(1)} = \frac{1}{2} \left(\nabla_\rho \nabla_\mu h_\nu{}^\rho + \nabla_\rho \nabla_\nu h_\mu{}^\rho - \nabla^2 h_{\mu\nu} - \nabla_\mu \nabla_\nu h \right), \quad (4.4)$$

$$R^{(1)} = \nabla_\mu \nabla_\nu h^{\mu\nu} - \nabla^2 h - h^{\mu\nu} R_{\mu\nu}^{(0)}, \quad (4.5)$$

where ∇ is the covariant derivative with respect to the unperturbative AdS background $g_{\mu\nu}^{(0)}$. In the right hand side of (4.3), $T_{\mu\nu}^{(1)}$ denotes the linearized terms of $T_{\mu\nu}^{\text{Weyl}}$ in (2.6). The explicit expression for $T_{\mu\nu}^{(1)}$ is given in Appendix (A).

It is obvious that the linearized wave equation (4.3) in general includes derivative terms with order higher than two which may make it difficult to study the propagating graviton. Our strategy of finding a second order differential equation is as follows. We first note that higher order (greater than 2) derivatives of the metric fluctuations come from sources $\square R^{(1)}$, $\nabla_\mu \nabla_\nu R^{(1)}$ and $\square R_{\mu\nu}^{(1)}$ in (A.4)-(A.6) in appendix (A). It follows from equation (4.3) that one can replace $R^{(1)}$ and $R_{\mu\nu}^{(1)}$ approximately by their leading order values by taking $\gamma \rightarrow 0$,

$$R^{(1)} \simeq \frac{R^{(0)} - 2\Lambda}{2 - D} h \quad (4.6)$$

$$R_{\mu\nu}^{(1)} \simeq \frac{1}{2} \left(\frac{R^{(0)} - 2\Lambda}{2 - D} \right) g_{\mu\nu}^{(0)} h + \frac{1}{2} R^{(0)} h_{\mu\nu} - \Lambda h_{\mu\nu}. \quad (4.7)$$

In this way all derivatives become the second order

$$\square R^{(1)} = \frac{1}{2-D} \left(\square R^{(0)} h + 2\nabla_\mu R^{(0)} \nabla^\mu h + (R^{(0)} - 2\Lambda) \square h \right) \quad (4.8)$$

$$\nabla_\mu \nabla_\nu R^{(1)} = \frac{1}{2-D} \left(\nabla_\mu \nabla_\nu R^{(0)} h + \nabla_\mu R^{(0)} \nabla_\nu h + \nabla_\nu R^{(0)} \nabla_\mu h + (R^{(0)} - 2\Lambda) \nabla_\mu \nabla_\nu h \right) \quad (4.9)$$

$$\begin{aligned} \square R_{\mu\nu}^{(1)} &= \frac{g_{\mu\nu}^{(0)}}{4-2D} \left(\square R^{(0)} h + 2\nabla_\mu R^{(0)} \nabla^\mu h + (R^{(0)} - 2\Lambda) \square h \right) + \\ &+ \frac{1}{2} \left(\square R^{(0)} h_{\mu\nu} + 2\nabla_\mu R^{(0)} \nabla^\mu h_{\mu\nu} + R^{(0)} \square h_{\mu\nu} \right) - \Lambda \square h_{\mu\nu}. \end{aligned} \quad (4.10)$$

And the linearized equation of motion can be obtained by substituting the perturbative metric.

Generally speaking, there are three types of metric perturbations: scalar perturbation, vector perturbation and tensor perturbation. In the present paper, we shall work in the $h_{u\mu} = 0$ gauge and use the Fourier decomposition

$$h_{\mu\nu}(t, z, r) = \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + ikz} h_{\mu\nu}(k, r),$$

where we choose the momenta to be along the z -direction, and for the sake of simplicity we have fixed $D = 5$ as an example ².

In this gauge, one can categorize the metric perturbations to the following three types according to the $O(2)$ rotation in the (x, y) -plane [41]:

- tensor type (scalar channel): $h_{xy} \neq 0$, $h_{xx} = -h_{yy} \neq 0$, (others) = 0
- vector type (shear channel): $h_{xt} \neq 0$, $h_{xz} \neq 0$, (others) = 0
(equivalently, $h_{yt} \neq 0$, $h_{yz} \neq 0$, (others) = 0)
- scalar type (sound channel): $h_{tz} \neq 0$, $h_{tt} \neq 0$, $h_{xx} = h_{yy} \neq 0$, and $h_{zz} \neq 0$, (others) = 0

In what follows we mainly focus on the first two kinds of perturbations, leaving the scalar type for future work.

4.1 Tensor-type perturbation

In this subsection we pay our attention to the tensor-type perturbation. A nontrivial equation of motion in (4.3) is coming from (x, y) component. In particular, we mainly focus on the small metric fluctuation $h_y^x(t, z, u) \equiv \phi(t, z, u)$ around the AdS black brane background(3.10) of the form

$$ds^2 = \frac{-f(u)dt^2 + d\vec{x}^2 + 2\phi(t, z, u)dxdy}{L^2 z_0^2 u} + \frac{L^2 du^2}{4u^2 f(u)}. \quad (4.11)$$

²Hereafter we will pay our attention to the case with $D = 5$. Higher dimensions will be investigated in the future.

According to the AdS/CFT correspondence, the fluctuation field $\phi(t, z, u)$ corresponds to the component T_{12} of the energy-momentum tensor of the boundary theory.

After using Fourier decomposition

$$\phi(t, z, u) = \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + i k z} \phi(k, u),$$

we can obtain the following linearized equation of motion for $\phi(u)$ from the equation (4.3) for $D = 5$:

$$\begin{aligned} 0 = \phi''(u) + \left(\frac{g'_T(u) + 32\kappa\gamma u^{3/2} f(u^{-1/2} f''(u))'}{g_T(u)} \right) \phi'(u) \\ + \frac{\bar{\omega}^2}{u f^2(u)} \phi(u) - \frac{\bar{k}^2 C_k(u)}{u^2 g_T(u)} \phi(u) + I(u) \phi(u), \end{aligned} \quad (4.12)$$

where

$$\begin{aligned} g_T(u) &\equiv u^{-1} f(u) [3L^2 - \kappa\gamma (96 - 136f(u) + 74u f'(u) - 20u^2 f''(u))], \\ C_k(u) &= 3L^2 - \kappa\gamma (96 - 136f(u) + 106u f'(u) - 28u^2 f''(u)), \\ \bar{\omega} &\equiv \frac{z_0 L^2}{2} \omega, \quad \bar{k} \equiv \frac{z_0 L^2}{2} k, \end{aligned}$$

and $I(u)$ is a term independent of ω , k , $\phi(u)$ and its derivatives. The explicit expression of $I(u)$ is not important in our following discussion.

4.2 Vector-type perturbation

In this case, nonzero small metric fluctuations are $h_{xt}(x) \neq 0$, $h_{xz}(x) \neq 0$. For convenience, we set $h_t^x(u) = g^{(0)xx} h_{xt}(u) = \phi(u)$, $h_z^x(u) = g^{(0)xx} h_{xz}(u) = \psi(u)$. Equivalently, the perturbative metric is given by

$$ds^2 = \frac{-f(u)dt^2 + d\vec{x}^2 + 2\phi(t, z, u)dxdt + 2\psi(t, z, u)dxdz}{L^2 z_0^2 u} + \frac{L^2 du^2}{4u^2 f(u)}. \quad (4.13)$$

Nontrivial equations in the equation of motion (4.3) appear from (t, x) , (u, x) and (x, z) components. It is not difficult to show that only two of these three linearized equations are independent. Without loss of generality in the present paper we choose equations coming from the (t, x) and (u, x) components. After making Fourier decompositions

$$\begin{aligned} \phi(t, z, u) &= \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + i k z} \phi(k, u), \\ \psi(t, z, u) &= \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + i k z} \psi(k, u), \end{aligned}$$

they are in general of the following forms:

$$0 = \phi''(u) + A_1(u)\phi'(u) + \left(A_2(u)\omega^2 + A_3(u)k^2 + A_4(u) \right) \phi(u) + A_5(u)k\omega\psi(u), \quad (4.14)$$

$$\begin{aligned} 0 = \omega\phi'(u) + k f(u)\psi'(u) + \kappa\gamma \left[B_1(u)\omega\phi'(u) + B_2(u)k\psi'(u) \right. \\ \left. + B_3(u)\omega\phi(u) + B_4(u)k\psi(u) \right], \end{aligned} \quad (4.15)$$

where $A_i(u)$ and $B_i(u)$ are some functions of u . Explicit expressions for $A_2(u)$, $A_3(u)$, $A_5(u)$, $B_1(u)$ and $B_2(u)$, are given in Appendix (B). Expressions for $A_1(u)$, $A_4(u)$, $B_3(u)$ and $B_4(u)$ are irrelevant to our discussion below where we only focus on the large momenta limit.

Now we can combine the above set of equations into one third order differential equation for $\phi(u)$. This can be achieved by noting that $\psi(u)$ in (4.15) has a perturbative solution in terms of $\phi(u)$ as we expand it to the order of $\mathcal{O}(\gamma^2)$,

$$\psi'(u) = -\frac{\omega}{kf(u)}\phi'(u) + \kappa\gamma \left[\left(-\frac{\omega B_1(u)}{kf(u)} + \frac{\omega B_2(u)}{kf(u)^2} \right) \phi'(u) + \dots \right] + \mathcal{O}(\gamma^2), \quad (4.16)$$

where “...” denotes terms that are irrelevant to our final results with the large momenta limit. Differentiating (4.14) with respect to u and substituting (4.16) into it we obtain

$$0 = \phi'''(u) + \tilde{A}_2(u)\omega^2\phi'(u) + A_3(u)k^2\phi'(u) + \dots + \mathcal{O}(\gamma^2), \quad (4.17)$$

where again “...” are irrelevant terms to our results and

$$\tilde{A}_2(u) = A_2(u) - \frac{A_5(u)}{f(u)} + \frac{3z_0^2 L^6 (B_2(u) - B_1(u)f(u))}{4uf(u)^3 g_V(u)} \kappa\gamma, \quad (4.18)$$

with $g_V(u)$ given in (B.4).

5 Causality Violation

It was shown that the causality could be violated if higher derivative terms are considered [31–35]. In this section we analyze the causality violation of Einstein-Weyl gravity and obtain the new constraint imposed on the coupling γ of the theory.

5.1 Tensor-type perturbation

We rewrite the wave function as

$$\phi(x, u) = e^{-i\omega t + ikx_3 + ik_u u}, \quad (5.1)$$

and take large momenta limit $k^\mu \rightarrow \infty$. In the end, one can find that the equation of motion (4.12) reduces to

$$k^\mu k^\nu g_{\mu\nu}^{\text{eff}} \simeq 0, \quad (5.2)$$

where the effective metric is given by

$$ds_{\text{eff}}^2 = g_{\mu\nu}^{\text{eff}} dx^\mu dx^\nu = \frac{f(u)}{L^2 z_0^2 u} \left(-dt^2 + \frac{1}{c_2^2} dx_3^2 \right) + \frac{L^2}{4u^2 f(u)} du^2. \quad (5.3)$$

Note that c_2^2 can be interpreted as the local speed of helicity-2 graviton:

$$c_2^2(u) = \frac{f^2(u) C_k(u)}{u g_T(u)}. \quad (5.4)$$

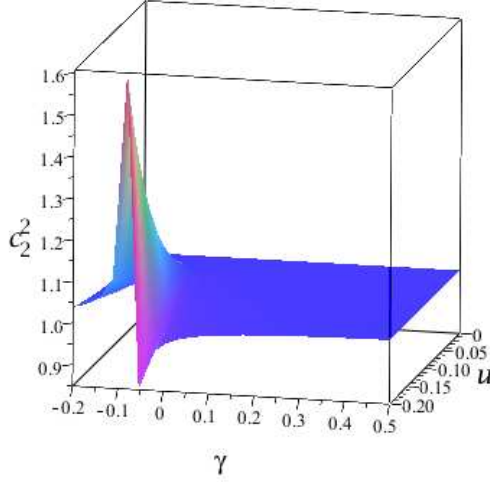


Figure 1: Local speed of helicity two modes c_2^2 as a function of u with different γ for fixed $\kappa = 1$ and $L = 1$.

Inserting $g_T(u)$ and $C_k(u)$ into above formula, we can expand c_2^2 near the boundary $u = 0$,

$$c_2^2 - 1 = \left(\frac{8\kappa\gamma - 3L^2}{40\kappa\gamma + 3L^2} \right) u^2 + \mathcal{O}(u^3). \quad (5.5)$$

It was shown in [32] that for Gauss-Bonnet gravity the causality violation happens when $c_2 > 1$, i.e., the coefficient of u^2 is positive. This is because the geodesics which starts at spatial infinity (the boundary) can bounce back to the boundary. From AdS/CFT correspondence, it was pointed out that the superluminal graviton propagation corresponds to the superluminal propagation of metastable quasiparticles in the boundary CFT which leads to the microcausality violation in the boundary CFT [32]. Similarly, for NEW gravity, the causality violation appears when

$$\left(\frac{8\kappa\gamma - 3L^2}{40\kappa\gamma + 3L^2} \right) > 0,$$

or equivalently

$$\kappa\gamma > \frac{3L^2}{8} \quad \text{or} \quad \kappa\gamma < -\frac{3L^2}{40}. \quad (5.6)$$

Our numerical plot (1) gives a visualized picture of the relationship between local speed of helicity two modes and u with different value of coupling γ .

5.2 Vector-type perturbation

Similar to the above causality analysis of tensor modes, we rewrite the wave function as

$$\phi(x, u) = e^{-i\omega t + ikx_3 + ik_u u}, \quad (5.7)$$

and take large momenta limit $k^\mu \rightarrow \infty$. The equation (4.17) then becomes

$$k_u^2 + \tilde{A}_2(u)\omega^2 + A_3(u)k^2 \simeq 0. \quad (5.8)$$

Then it is straightforward that the local speed of helicity one modes is

$$c_1^2 = \frac{\omega^2}{k^2} = -\frac{A_3(u)}{\tilde{A}_2(u)}. \quad (5.9)$$

This leads to

$$c_1^2 - 1 = -\left(\frac{3L^2 + 128\kappa\gamma}{3L^2 + 104\kappa\gamma}\right)u^2 + \mathcal{O}(u^3). \quad (5.10)$$

Therefore the microcausality violation appears in the boundary CFT for

$$\frac{3L^2 + 128\kappa\gamma}{3L^2 + 104\kappa\gamma} < 0,$$

or equivalently

$$-\frac{3L^2}{104} < \kappa\gamma < -\frac{3L^2}{128}. \quad (5.11)$$

Our numerical result confirms this argument as shown in figure (2).

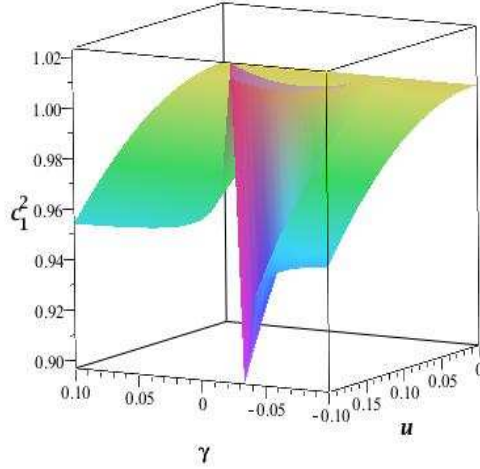


Figure 2: Local speed of helicity one modes c_1^2 as a function of u with different γ for fixed $\kappa = 1$ and $L = 1$.

6 Conclusions and Discussions

In this paper, we have shown when small metric fluctuations around an AdS black brane in NEW gravity are considered, the propagations of the linearized modes impose a new

bound on the coupling of the Weyl-squared terms. Our analysis in the present paper found the constraints for the scalar channel and shear channel are given, respectively, by

$$\text{scalar channel : } -\frac{3L^2}{40} \leq \kappa\gamma \leq \frac{3L^2}{8}, \quad (6.1)$$

$$\text{shear channel : } \kappa\gamma \leq -\frac{3L^2}{104} \text{ or } \kappa\gamma \geq -\frac{3L^2}{128}. \quad (6.2)$$

Combining the above two constraints with the tachyon-free condition (2.15) in five dimensions, one leads to the following bounds on the Weyl-squared coupling, i.e.,

$$-\frac{3L^2}{40} \leq \kappa\gamma \leq -\frac{3L^2}{104} \text{ or } -\frac{3L^2}{128} \leq \kappa\gamma \leq \frac{3L^2}{8}. \quad (6.3)$$

It is clear that the new bound imposes more stringent restrictions than that of the tachyon-free condition, improving predictive power of the theory.

It is expected that the sound channel would impose another bound on the coupling. The reason is that it was noted in [33, 34, 36] that for Gauss-Bonnet gravity, the upper bound of the Gauss-Bonnet coupling comes from the tensor-type perturbations, and it is nothing but the lower bound of the ratio of two central charges a/c in dual CFT. However, the lower bound, which comes from the scalar-type perturbations, corresponds precisely to the upper bound of a/c . Although it is not fully guaranteed to generalize this result to the NEW gravity, one still can expect that a similar result can be achieved. We will leave this issue for our future work.

In the present paper, we carry out the computations on the gravity side by considering small metric perturbations around the AdS black brane. It is of great interest to compute our new bound in the context of thermal CFTs, as what did in [36] and [33]. This kind of computation is important in that, on one hand, it may provide an effective check to our results, on the other hand, it may be helpful to our understanding about the whole picture.

We must emphasize that our calculations performed in this paper are approximately correct. To avoid solving fourth-order differential equations, we adopted an approximate solution twice by taking the small γ limit. Since the allowing value of γ as given in (6.3) is small enough, it is safe to make this approximation.

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A Expression for $T_{\mu\nu}^{(1)}$

In the expression (4.3), $T_{\mu\nu}^{(1)}$ is of the form

$$\begin{aligned}
T_{\mu\nu}^{(1)} = & \gamma \left\{ \frac{1}{(D-1)(D-2)} \left(h_{\mu\nu} R^{(0)2} + 2g_{\mu\nu}^{(0)} R^{(0)} R^{(1)} - 4R^{(1)} R_{\mu\nu}^{(0)} - 4R^{(0)} R_{\mu\nu}^{(1)} \right) - \right. \\
& - \frac{2}{D-2} \left[g_{\mu\nu}^{(0)} (R_{\rho\sigma} R^{\rho\sigma})^{(1)} + h_{\mu\nu} R_{\rho\sigma}^{(0)} R^{(0)\rho\sigma} - 2(D-2) (R^\rho{}_\mu R_{\rho\nu})^{(1)} \right] \\
& + \frac{2(D-3)}{(D-1)(D-2)} \left[g_{\mu\nu}^{(0)} (\square R)^{(1)} + h_{\mu\nu} \square R^{(0)} + (D-2) (\nabla_\mu \nabla_\nu R)^{(1)} - 2(D-1) (\square R_{\mu\nu})^{(1)} \right] \\
& + \frac{1}{2} g_{\mu\nu}^{(0)} \left(R_{\rho\sigma\lambda\kappa} R^{\rho\sigma\lambda\kappa} \right)^{(1)} + \frac{1}{2} h_{\mu\nu} R_{\rho\sigma\lambda\kappa}^{(0)} R^{(0)\rho\sigma\lambda\kappa} - \frac{4(D-4)}{D-2} (R^{\rho\sigma} R_{\mu\rho\nu\sigma})^{(1)} \\
& \left. - 2 (R_{\mu\rho\sigma\gamma} R_{\nu}{}^{\rho\sigma\gamma})^{(1)} \right\}. \tag{A.1}
\end{aligned}$$

where $R^{(0)}$, $R_{\mu\nu}^{(0)}$, $R_{\rho\sigma\lambda\kappa}^{(0)}$ and the covariant derivative are defined through the background metric $g_{\mu\nu}^{(0)}$. In (A.1) the following terms are given explicitly by

$$(R_{\rho\sigma} R^{\rho\sigma})^{(1)} = 2R^{(0)\mu\nu} R_{\mu\nu}^{(1)} - 2h^{\mu\rho} R^{(0)\nu}{}_\rho R_{\mu\nu}^{(0)}, \tag{A.2}$$

$$(R^\rho{}_\mu R_{\rho\nu})^{(1)} = R_{\rho\mu}^{(1)} R^{(0)\rho}{}_\nu + R^{(0)\rho}{}_\mu R_{\rho\nu}^{(1)} - h^{\rho\sigma} R_{\sigma\mu}^{(0)} R_{\rho\nu}^{(0)}, \tag{A.3}$$

$$(\square R)^{(1)} = \square R^{(1)} - h^{\mu\nu} \partial_\mu \partial_\nu R^{(0)} + \frac{1}{2} \nabla_\mu h \cdot \nabla^\mu R^{(0)}, \tag{A.4}$$

$$(\nabla_\mu \nabla_\nu R)^{(1)} = \nabla_\mu \nabla_\nu R^{(1)} - \frac{1}{2} (\nabla_\nu h_\mu^\rho + \nabla_\mu h_\nu^\rho - \nabla^\rho h_{\mu\nu}) \nabla_\rho R^{(0)}, \tag{A.5}$$

$$\begin{aligned}
(\square R_{\mu\nu})^{(1)} = & \frac{1}{2} \square R_{\mu\nu}^{(1)} + \frac{1}{2} (\nabla^\rho \nabla^\sigma h_{\rho\mu} - \square h_\mu^\sigma - \nabla^\rho \nabla_\mu h_\rho^\sigma) R_{\nu\sigma}^{(0)} \\
& + (\nabla^\sigma h_{\rho\mu} - \nabla_\rho h_\mu^\sigma - \nabla_\mu h_\rho^\sigma) \nabla^\rho R_{\nu\sigma}^{(0)} - \frac{1}{2} \nabla_\rho h^{\rho\sigma} \nabla_\sigma R_{\mu\nu}^{(0)} \\
& + \frac{1}{4} \nabla^\rho h \nabla_\rho R_{\mu\nu}^{(0)} - \frac{1}{2} h^{\rho\sigma} \nabla_\rho \nabla_\sigma R_{\mu\nu}^{(0)} + (\mu \leftrightarrow \nu), \tag{A.6}
\end{aligned}$$

$$(R_{\rho\sigma\lambda\kappa} R^{\rho\sigma\lambda\kappa})^{(1)} = 2R^{(0)\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}^{(1)} - 4h_\kappa^\mu R_{\mu\nu\rho\sigma}^{(0)} R^{(0)\kappa\nu\rho\sigma}, \tag{A.7}$$

$$(R^{\rho\sigma} R_{\mu\rho\nu\sigma})^{(1)} = R^{(0)\rho\sigma} R_{\mu\rho\nu\sigma}^{(1)} + R_{\rho\sigma}^{(1)} R^{(0)}{}_\mu{}^\rho{}_\nu{}^\sigma - h^{\rho\kappa} R^{(0)}{}_\kappa{}^\sigma \left(R_{\mu\rho\nu\sigma}^{(0)} + R_{\mu\sigma\nu\rho}^{(0)} \right), \tag{A.8}$$

$$\begin{aligned}
(R_{\mu\rho\sigma\gamma} R_{\nu}{}^{\rho\sigma\gamma})^{(1)} = & R_{\mu\rho\sigma\gamma}^{(1)} R^{(0)}{}_\nu{}^{\rho\sigma\gamma} + R_{\nu\rho\sigma\gamma}^{(1)} R^{(0)}{}_\mu{}^{\rho\sigma\gamma} - \\
& - R_{\mu\rho\sigma\gamma}^{(0)} \left(h_\kappa^\rho R^{(0)}{}_\nu{}^{\kappa\sigma\gamma} + h_\kappa^\sigma R^{(0)}{}_\nu{}^{\rho\kappa\gamma} + h_\kappa^\gamma R^{(0)}{}_\nu{}^{\rho\sigma\kappa} \right), \tag{A.9}
\end{aligned}$$

where

$$R_{\kappa\mu\sigma\nu}^{(1)} = h_{\rho\kappa} R^{(0)\rho}{}_{\mu\sigma\nu} + \frac{1}{2} [\nabla_\nu \nabla_\kappa h_{\mu\sigma} - \nabla_\nu \nabla_\mu h_{\kappa\sigma} - \nabla_\sigma \nabla_\kappa h_{\mu\nu} + \nabla_\sigma \nabla_\mu h_{\kappa\nu} + 2\nabla_{[\sigma} \nabla_{\nu]} h_{\kappa\mu}].$$

B Expressions for $A_i(u)$ and $B_i(u)$

Expressions for $A_i(u)$ ($i = 2, 3, 5$) are given respectively, by

$$A_2(u) = \frac{16\gamma\kappa z_0^2 L^4 (3 - 4f(u) - u^2 f''(u) + 3uf'(u))}{uf(u)^2 g_V(u)}, \quad (\text{B.1})$$

$$A_3(u) = \frac{z_0^2 L^4 (3L^2 + \gamma\kappa(68u^2 f''(u) - 218uf'(u) + 296f(u) - 192))}{4uf(u)g_V(u)}, \quad (\text{B.2})$$

$$A_5(u) = \frac{z_0^2 L^4 (3L^2 + \gamma\kappa(4u^2 f''(u) - 26uf'(u) + 40f(u)))}{4uf(u)g_V(u)}, \quad (\text{B.3})$$

where $g_V(u)$ is

$$g_V(u) = -3L^2 - \gamma\kappa (36u^2 f''(u) - 198uf'(u) + 296f(u) - 192). \quad (\text{B.4})$$

Expressions for $B_i(u)$ ($i = 1, 2$) are given, respectively, by

$$B_1(u) = -\frac{2(14u^2 f''(u) + 3uf'(u) - 20f(u))}{3L^2}, \quad (\text{B.5})$$

$$B_2(u) = -\frac{2(-2u^2 f''(u) + 13uf'(u) - 20f(u))}{3L^2}. \quad (\text{B.6})$$

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